

# Undoing the effect of loss on quantum entanglement

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**Entanglement distillation, the purpose of which is to probabilistically increase the strength and purity of quantum entanglement, is a primary element of many quantum communication and computation protocols. It is particularly necessary in quantum repeaters in order to counter the degradation of entanglement that inevitably occurs due to losses in communication lines. Here, we distil the Einstein-Podolsky-Rosen state of light, the workhorse of continuous-variable entanglement, using noiseless amplification. The advantage of our technique is that it permits recovering a macroscopic level of entanglement, however low the initial entanglement or however high the loss may be. Experimentally, we recover the original entanglement level after one of the Einstein-Podolsky-Rosen modes has experienced a loss factor of 20. The level of entanglement in our distilled state is higher than that achievable by direct transmission of any state through a similar loss channel. This is a key step towards realizing practical continuous-variable quantum communication protocols.**

Quantum technology protocols exploit the unique properties of quantum systems to fulfil communication, computing and metrology tasks that are impossible, inefficient or intractable for classical systems<sup>1</sup>. In many cases, the distribution of entanglement, correlations between subsystems that exceed those possible for classical systems, is a necessary condition for quantum technology protocols to succeed. However, entanglement is fragile and can easily be degraded by the communication or storage of the entangled systems. One solution to this problem is entanglement distillation<sup>2</sup>. Given an ensemble of weakly entangled quantum states, distillation techniques allow one to select or distil a smaller sub-ensemble of states that are more strongly entangled. This can be achieved using only local operations and classical communication. In this way, strong entanglement can be established between remote locations under conditions where it would be impossible without distillation (for example, with the losses that are common in quantum communication channels).

There are two broad classes of quantum optical technology protocols: those using quantum observables with a discrete spectrum, such as the spin of an electron, and those using quantum variables with a continuous spectrum, such as the position and momentum of a harmonic oscillator<sup>3</sup>. Our focus here is on the distillation of continuous-variable (CV) states. The primary entangled resource in CV systems is the two-mode squeezed vacuum state, also known as the Einstein-Podolsky-Rosen (EPR) state<sup>4</sup> because its idealized version was introduced by those scientists in the early days of quantum mechanics to illustrate quantum nonlocality.

The EPR state can be used to implement many quantum protocols, including continuous versions of teleportation and quantum key distribution<sup>5</sup>. An advantage of the CV approach to quantum communication is its universality: it is capable of transmitting arbitrary states of light, in contrast to the single-photon subspace of the Hilbert space to which the discrete method is limited. Furthermore, unlike their

discrete-variable counterparts, high-quality EPR states are readily available on demand at a high rate from parametric amplifiers.

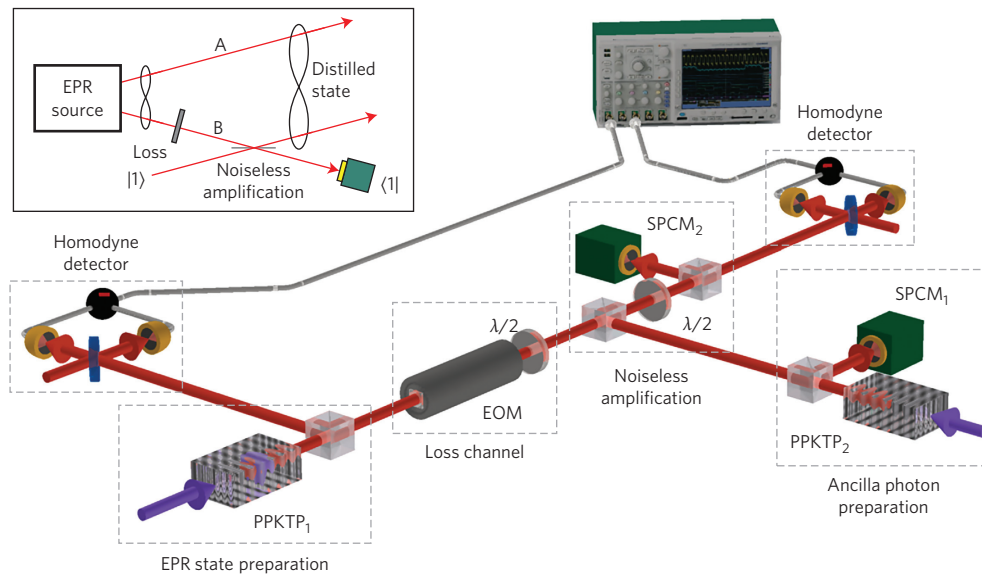
In application to quantum repeaters, those communication protocols that use single photons typically do not need a special distillation procedure to counter the effect of the losses. This is because if a photon is lost, it is not registered by the detector, so a loss event is automatically eliminated from further analysis. In CV protocols, quadrature detection occurs independently of the losses, so recovering an entangled resource suitable for use in a teleportation or repeater protocol requires a dedicated distillation step.

In this Article, we present experimental results demonstrating the distillation of optical CV entanglement in two settings: (1) for very low initial squeezing and (2) after transmission through a lossy channel. In the second setting, we directly observe an entanglement strength of our distilled state that exceeds anything possible via deterministic transmission of the states through the same channel. That is, even if a perfectly pure, infinitely entangled EPR state were passed through that channel, the resulting entanglement would be inferior to what we observe for our distilled state. We will refer to this as breaking the deterministic bound.

Our protocol relies on the technique of noiseless linear amplification (NLA)<sup>6</sup>, in contrast to previous CV entanglement distillation demonstrations based on photon subtraction<sup>7,8</sup>. Photon subtraction is unable to enhance entanglement in the EPR state by more than a factor of two, which is by far insufficient to compensate for a loss occurring in a typical communication line. NLA does not suffer from this limitation, and in principle allows the entanglement to be restored to a macroscopic level after an arbitrarily high loss<sup>9</sup>. It is this feature of NLA that enables us to break the deterministic bound. It represents a major step forward in realizing protocols that can enhance quantum technologies under practical conditions.

A key feature of our experiment is that heralded, free-propagating distilled EPR states are produced by our protocol. This differs from a

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**Figure 1 | Experimental set-up.** Inset: Conceptual scheme. The NLA distillation event corresponds to the click in the single-photon detector placed in the beamsplitter output.

previous experiment<sup>10</sup>, where NLA was emulated by post-selecting the quadrature measurements of an asymmetrically attenuated EPR state. That virtual distillation protocol cannot herald free-propagating EPR pairs and hence cannot be used for quantum teleportation or repeaters.

**Concept**

The conceptual scheme of the present set-up is shown in Fig. 1, inset. We explain the idea of our method for the practically relevant case of low initial squeezing and high losses. The initial EPR state prepared in a pair of modes A and B is then given by

$$|\Psi_{AB}\rangle = |00\rangle - \gamma|11\rangle \tag{1}$$

where we have limited the analysis to the first order of the squeezing amplitude  $\gamma \ll 1$ . A loss channel with amplitude transmissivity  $\tau \ll 1$  applied to mode B will degrade the non-vacuum component of this state, transforming it into a mixture

$$\hat{\rho} = (|00\rangle - \gamma\tau|11\rangle)(\langle 00| - \gamma\tau\langle 11|) + \gamma^2(1 - \tau^2)|10\rangle\langle 10|$$

We then subject this state to NLA. In contrast to previous NLA experiments<sup>11–15</sup>, where either the method of generalized quantum scissors<sup>16,17</sup> or coherent superposition of photon subtraction and addition was used, here we employ a technique known as quantum catalysis<sup>18</sup>. The mode to be amplified is reflected from a low-reflectivity beamsplitter, on which it is brought into interference with an ancillary single photon. The amplification event is heralded by detecting a single photon in the other output port of that beamsplitter (Fig. 1, inset). This will happen either if (1) the ancilla photon is reflected (the probability amplitude of this event equals the amplitude reflectivity  $r \ll 1$  of the beamsplitter) or (2) a photon from the EPR state is transmitted (this occurs with probability amplitude  $\gamma\tau$ ). In the latter case, the ancilla photon is likely to transmit through the beamsplitter into the reflected channel of the EPR state. If these possibilities are made indistinguishable by proper mode matching of the ancilla photon and the EPR state, their amplitudes add coherently. As a result, the EPR state is transformed into

$$\hat{\rho}' = (r|00\rangle - \gamma\tau|11\rangle)(r\langle 00| - \gamma\tau\langle 11|) + r^2\gamma^2(1 - \tau^2)|10\rangle\langle 10| \tag{2}$$

The second term in the above state is of higher order in  $\gamma$ ,  $\tau$  and  $r$  than the first term, so the final state is well approximated by pure superposition:

$$|\Psi'_{AB}\rangle = r|00\rangle - \gamma\tau|11\rangle \tag{3}$$

We see that the reduction of the single-photon component due to low initial squeezing and/or loss in the communication is compensated by the reduction of the vacuum part due to the catalysis. This is equivalent to NLA with an amplitude gain of  $g = 1/r$ .

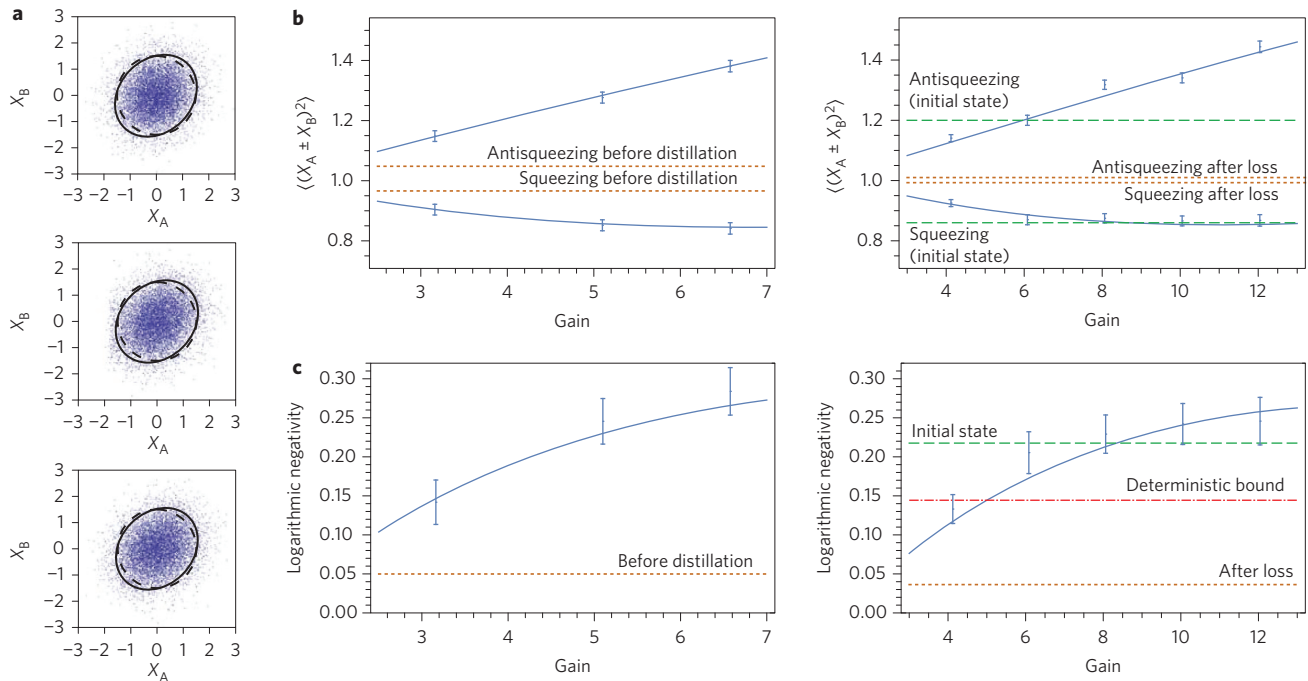
The amount of entanglement to be recovered depends on the ratio  $r/\gamma\tau$  and reaches one ebit at  $g = (\gamma\tau)^{-1}$ . If we instead aim to optimize the two-mode squeezing of the resulting state, this goal is reached at a slightly lower gain of  $g = [\gamma\tau(1 + \sqrt{2})]^{-1}$ , resulting in a two-mode squeezing factor of  $2 - \sqrt{2} \approx 0.59$  with respect to the standard quantum limit. The precise value of the optimal gain may vary in dependence on the efficiency of the ancilla photon source as well as the chosen continuous-variable entanglement measure. The bottom line is, though, that however high the loss, and however low the initial squeezing may be, one can choose the NLA gain to recover a macroscopic amount of entanglement in the distilled state (details of the theoretical calculations are provided in Supplementary Section II). Importantly, the effective implementation of the catalysis procedure in the practically important limit  $\gamma, \tau \ll 1$  does not require number-discriminating photon detectors, and the fidelity of the distilled state is not significantly affected by the quantum efficiency of that detector.

Our method is related to the idea of Mićuda and colleagues<sup>19</sup>, who proposed to counter the effect of losses in a communication line by applying NLA with gains below and above unity before and after the transmission, respectively. Optimized specifically to the EPR state, this protocol would become equivalent to ours.

It is also instructive to compare our work with that of Ourjoumtsev and co-workers<sup>20</sup>, who obtained the nonlocal single-photon state by nonlocal photon subtraction from the two modes of the EPR state. The advantage of our method is that it only uses local operations and classical communications and is able to counter losses.

**Experiment**

The EPR state (Fig. 1) was prepared in a periodically poled potassium titanyl phosphate crystal (PPKTP<sub>1</sub>) where type II parametric



**Figure 2 | Experimental results.** **a**, A total of 10,000 samples of raw position quadrature data for the original two-mode squeezed state (top) and the distilled state after applying the two methods of degrading the entanglement (reducing the initial squeezing, middle; asymmetric loss, bottom). The NLA gain is 6.5 for the middle plot and 10 for the bottom. The degree of correlation is similar in all three cases. The quadratures exhibit non-classical correlations at a level below the shot noise (dashed circle). **b,c**, Analysis of the distilled states as a function of NLA gain. Left column: the case of reduced initial squeezing. Right column: the case of asymmetric loss. In **b**, two-mode squeezing is displayed, measured by the variances of the sum (anti-squeezing) and difference (squeezing) of the position quadratures in the two channels of the distilled EPR state. In **c**, the logarithmic negativity is shown. The vertical axes in **b** are scaled in units of standard quantum limit. The theoretical curves for the case of low initial squeezing are calculated assuming the initial squeezing parameter of  $\gamma = 0.05$ , detection efficiencies in the undistilled and distilled channels of  $\eta_A = 0.5$  and  $\eta_B = 0.5$ , respectively, and a single-photon preparation efficiency of  $\eta = 0.65$  (refs 15,28,29). For the case of a loss channel,  $\gamma = 0.135$  and  $\eta_A = 0.45$ ; other parameters are the same. The efficiency parameters are defined in Supplementary Fig. 2.

down-conversion was taking place. The crystal was pumped with frequency-doubled pulses at 390 nm, generated by a Ti:sapphire laser with a repetition rate of 76 MHz and a pulse width of  $\sim 1.8$  ps. The outgoing orthogonally polarized light modes were spatially split on a polarizing beamsplitter. Each mode was subjected to balanced homodyne detection<sup>21</sup>. The phase of one of the local oscillator phases was varied using a piezoelectric transducer (PZT). This allowed us to obtain a quorum of the two-mode quadrature measurements and therefore completely characterize the state. We observed the variances in the difference and sum of the position quadratures of the initial state to be  $\langle (X_A - X_B)^2 \rangle = 0.86 \pm 0.01$  and  $\langle (X_A + X_B)^2 \rangle = 1.20 \pm 0.01$ , measured in units of the vacuum state level (Fig. 2a). This corresponds to the squeezing parameter of  $\gamma = 0.18 \pm 0.04$ .

Before distillation, the entanglement of the initial state was degraded by one of two methods. In the first, we turn the polarization of the pump beam by  $\theta = 76 \pm 2^\circ$ , resulting in the reduction of the squeezing parameter  $\gamma$  by a factor of  $\cos\theta = 0.22 \pm 0.02$ . This reduces the EPR correlations, bringing the difference and sum quadrature noise levels to 0.966 and 1.044, respectively. The second method consists of introducing asymmetric attenuation, imitating the propagation of mode 2 through a lossy optical channel with an intensity transmissivity of  $\tau^2 = 0.05$ . The difference and sum quadrature noise levels then become 0.993 and 1.010, respectively. Such low squeezing levels could not be reliably measured by homodyne detection, so the variance values quoted above are based on the measured initial squeezing and the known attenuation factor.

The single photon state required for the NLA was obtained from parametric down-conversion in another crystal, PPKTP<sub>2</sub>, at a rate of

50–100 kHz, heralded by the clicks of a single-photon counting module (SPCM<sub>1</sub>)<sup>22</sup>. The variable beamsplitter that enacts the NLA was implemented by a combination of two polarizing beamsplitters and a half-wave plate between them. One output of that beamsplitter is directed onto another photon detector (SPCM<sub>2</sub>) and the other is subjected to homodyne detection. The distillation events correspond to both SPCMs firing synchronously and occur at a rate of  $\sim 10$ –100 Hz.

To characterize the distilled state, the quadrature data from both detectors should be accompanied by knowledge of the sum  $\varphi$  of the local oscillator phases for each data sample<sup>8</sup>. We acquired this information by using the fraction of mode B of the original EPR state that was reflected from the NLA beamsplitter towards the homodyne detector in the absence of the heralding event. This field exhibits phase-dependent quadrature correlations with mode A. Because it is supplied at the master laser repetition rate, we observed a sufficiently clear correlation signal to retrieve  $\varphi$  at any moment in time. It is the availability of this phase evaluation technique that motivated us to choose the catalysis scheme over the more common scissors scheme for the NLA.

To mitigate the degradation of this reference signal by the loss channel, we implemented that channel using an electro-optical modulator (EOM) followed by a half-wave plate. The EOM was controlled by a square wave with a period of 0.2 s (during which the phase was assumed to vary linearly) and a duty cycle of 0.35. The ‘off’ period, when the transmission through the EOM was maximized, was used to measure  $\varphi$ . During the ‘on’ period, the EOM voltage was set to attenuate the propagating signal and the quadrature samples associated with the distilled EPR state were acquired.

Note that this approach would not work in practical quantum communication, as there would be no possibility to switch the loss off. Instead, the sending party could periodically apply constant phase space displacement to mode B. This displacement would then be measured by the receiving party's homodyne detector, thereby providing the required phase information.

## Results and discussion

The experimental results are summarized in Fig. 2. Figure 2a presents samples of the raw data for the original and distilled states, while the left and right columns in Fig. 2b,c correspond to the distilled states after applying the two methods of degrading the entanglement. Figure 2b shows the variances of the sum and difference of the position quadratures (the behaviour of the momentum quadrature is similar to that of the position). The amount of recovered squeezing is maximized at  $g = 6-7$  for the first method and at  $g = 10-12$  for the second, which is consistent with the theoretical expectation in the ideal case  $g = [\gamma r(1 + \sqrt{2})]^{-1}$ . In both cases, the two-mode squeezing of the state recovered by distillation is at the same level as the initial state, and dramatically higher than that of the EPR state with reduced squeezing, or the attenuated EPR state before distillation. However, the observed two-mode squeezing does not reach the above-mentioned theoretical limit of 0.59 because of various experimental inefficiencies such as those involved in preparing the EPR state and the ancilla photon, mode matching on the NLA beamsplitter and detection inefficiencies.

We also observed that the uncertainty product  $\langle(X_A - X_B)^2\rangle \langle(X_A + X_B)^2\rangle$  for the quadrature sum and difference in the two modes typically exceeds that for the initial state. This happens primarily because the distilled state (3) is non-Gaussian and is not indicative of that state being less pure than the initial one.

To quantitatively estimate the degree of entanglement at different stages of our experiment, we used the logarithmic negativity<sup>23</sup>. The value of this parameter in the distilled state is a factor of about five greater than that in the degraded EPR states (Fig. 2c). To our knowledge, this factor of entanglement increase achieved in a distillation protocol is higher than anything reported to date, not only in the continuous-variable domain, but also in the discrete-variable domain<sup>7,8,24</sup>.

For distillation after asymmetric loss, we can also compare this parameter with that of the perfect EPR state that has experienced the same loss, in which case  $E_n = -\log_2((1 - \tau^2)/(1 + \tau^2)) = 0.144$  (see Supplementary Section I for details). The entanglement observed in the distilled state is consistently stronger than this benchmark, demonstrating that our method is able to overcome the deterministic bound.

It is instructive to estimate our distillation procedure by finding a hypothetical 'equivalent' EPR state that, after appropriate attenuation of both channels, would exhibit the same quadrature statistics as our distilled state<sup>25</sup>. In Supplementary Section III we find the squeezing parameter of that state and the equivalent efficiency of the loss channel in mode B to be, respectively, about two and three times greater than those in the actual experiment. This illustrates the capability of our procedure to largely compensate for the losses and low initial squeezing.

## Conclusion

In comparison with other CV entanglement distillation methods, the procedure demonstrated here is particularly effective in the regimes of low initial squeezing and/or high loss in the transmission channel<sup>9</sup>. It allows recovery of entanglement after the loss, however significant it may be. This property makes our method promising for the construction of a CV quantum error correction protocol<sup>25</sup> and hence ultimately a CV quantum repeater.

The distilled state (3) does not contain a significant fraction of terms beyond the single-photon level. This imposes a limitation

on the amount of squeezing and entanglement attainable through the distillation. One can envision a number of ways to address this. In particular, the non-Gaussian state (3) can be further distilled using the Gaussian procedure of ref. 26, which can be implemented using only four quantum optical memory cells<sup>27</sup>. By applying this protocol in an iterative manner, an infinitely squeezed EPR state can in principle be obtained. Alternatively, one can think of combining a chain of states (3) in a quantum repeater directly by means of Bell measurements in a continuous-variable basis, which will deterministically produce an entangled state of single-rail optical qubits.

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### Author contributions

The experiment was conceived and designed by A.E.U., I.F., Y.K., T.C.R. and A.L., and performed by A.E.U., I.F., Y.K., A.A.P. and A.L. The data were analysed by A.E.U., I.F. and A.L. A.E.U., I.F., Y.K., T.C.R. and A.L. contributed to writing the paper.

### Additional information

Supplementary information is available in the [online version](#) of the paper. Reprints and permissions information is available online at [www.nature.com/reprints](http://www.nature.com/reprints). Correspondence and requests for materials should be addressed to A.L.L.

### Competing financial interests

The authors declare no competing financial interests.