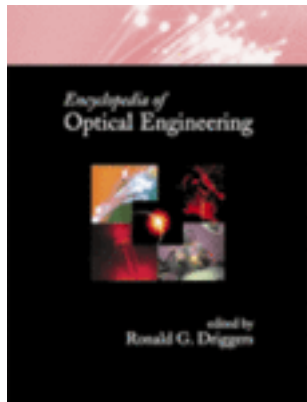


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### Fresnel Equations

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# Fresnel Equations

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## Abstract

The Fresnel equations, which determine the reflection and transmission of light incident on an interface of two media with different indices of refraction, are among the most fundamental findings of classical optics. This entry offers a detailed derivation of the equations and discusses some of their major consequences (in particular, Brewster effect, total internal reflection, and the Goos-Hänchen shift), as well as applications both in everyday optics and in specialized equipment.

## INTRODUCTION

The Fresnel equations relate the amplitudes, phases, and polarizations of the transmitted and reflected waves that emerge when light enters an interface between two transparent media with different indices of refraction, to the corresponding parameters of the incident waves. These equations were derived by Augustin-Jean Fresnel in 1823 as a part of his comprehensive wave theory of light. However, the Fresnel equations are fully consistent with the rigorous treatment of light in the framework of Maxwell equations.

The Fresnel equations are among the most fundamental findings of classical optics. Because they describe the behavior of light at optical surfaces, they are relevant to virtually all fields of optical design: lens design, imaging, lasers, optical communication, spectroscopy, and holography. Good understanding of the principles behind Fresnel equations is necessary in designing optical coatings and Fabry-Perot interferometers.

This entry begins with a detailed derivation of the Fresnel equations based on Snell's law and the boundary relations for the electric and magnetic fields at an interface between two media with different electromagnetic properties. We then proceed to discuss the primary consequences of these equations, such as intensity reflectivities and Brewster's effect. The final section of the entry is dedicated to numerous applications of the Fresnel equations.

## DERIVATION

To derive the Fresnel equations, consider two optical media separated by an interface, as shown in Fig. 1. A plane optical wave is propagating toward the interface with wave vector  $\vec{k}_i$  oriented at angle  $\theta_i$  with respect to the interface normal. The electric field amplitude of the wave is given by  $E_i$ .

On incidence onto the interface, this wave will be partially transmitted and partially reflected. The transmitted

wave will propagate at angle  $\theta_t$  which is determined by Snell's law:

$$\frac{\sin \theta_i}{\sin \theta_t} = \frac{n_2}{n_1} \quad (1)$$

where  $n_1$  and  $n_2$  are the refractive indices of the two media. The angle  $\theta_r$  of the reflected wave is equal to  $\theta_i$  according to the law of reflection. We denote the amplitudes of these two waves as  $E_i$  and  $E_r$ , respectively. Our goal is to determine these amplitudes.

To accomplish this, we apply the boundary conditions for the electric and magnetic fields at an interface between two media with different electromagnetic properties, which are known from electrostatics. Specifically, the components of the electric field  $\vec{E}$  and magnetic field  $\vec{H}$ , which are tangent to the surface, must be continuous across the boundary.

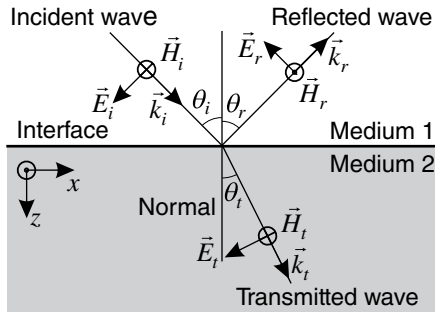
Because the electromagnetic wave is transverse, the field incident onto the interface can be decomposed into two polarization components, one P-polarized, i.e., with the electric field vector inside the plane of incidence, and the other one S-polarized, i.e., orthogonal to that plane. (Under the plane of incidence, we understand the plane that is formed by the vector  $\vec{k}_i$  and the normal to the interface.) We will derive the Fresnel equations for these two cases separately.

We begin by concentrating on the case when the incident wave is P-polarized (Fig. 1). Due to symmetry, the transmitted and reflected waves will have the same polarization. Because the  $\vec{E}$ ,  $\vec{H}$ , and  $\vec{k}$  vectors must form a right-handed triad for each of the waves, the directions of all field vectors are uniquely defined up to a sign convention, which is chosen as illustrated in Fig. 1. The boundary condition for the electric field then becomes:

$$E_i \cos \theta_i + E_r \cos \theta_i = E_t \cos \theta_t \quad (2)$$

For the magnetic field, which is collinear in all three waves, this condition takes the form:

$$H_i - H_r = H_t \quad (3)$$



**Fig. 1** Field vectors of the incident, transmitted, and reflected waves in case the electric field vectors lie within the plane of incidence (P polarization).

To solve these equations, we need to incorporate the relation between the electric and magnetic field amplitudes for each wave. We know from Maxwell equations that these amplitudes in any plane electromagnetic wave must satisfy

$$H = \sqrt{\frac{\epsilon}{\mu}} E \quad (4)$$

where  $\epsilon$  and  $\mu$  are the electric permittivity and magnetic permeability, respectively, of the material in which the wave propagates. Since the index of refraction of a material is given by  $n = c \sqrt{\epsilon\mu}$ , we have:

$$H_{i,r} = n_1 E_i / c \text{ and } H_t = n_2 E_t / c \quad (5)$$

and thus, from Eq. (3),

$$n_1(E_i - E_r) / \mu_1 = n_2 E_t / \mu_2 \quad (6)$$

Combining Eqs. 2 and 6, we arrive at the Fresnel equations for the P-polarized wave:

$$r_p = \frac{(n_1/\mu_1) \cos \theta_i - (n_2/\mu_2) \cos \theta_t}{(n_1/\mu_1) \cos \theta_i + (n_2/\mu_2) \cos \theta_t} \quad (7)$$

$$t_p = \frac{2(n_1/\mu_1) \cos \theta_i}{(n_1/\mu_1) \cos \theta_i + (n_2/\mu_2) \cos \theta_t} \quad (8)$$

where we defined the amplitude reflection and transmission coefficients:

$$r = \frac{E_r}{E_i} \text{ and } t = \frac{E_t}{E_i} \quad (9)$$

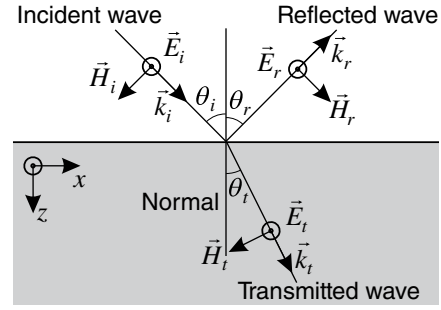
In the case of S polarization (Fig. 2), in much the same way, we write the boundary conditions as

$$E_i + E_r = E_t \quad (10)$$

$$-H_i \cos \theta_i + H_r \cos \theta_i = -H_t \cos \theta_t \quad (11)$$

from which we derive the second pair of Fresnel equations:

$$r_s = \frac{(n_1/\mu_1) \cos \theta_i - (n_2/\mu_2) \cos \theta_t}{(n_1/\mu_1) \cos \theta_i + (n_2/\mu_2) \cos \theta_t} \quad (12)$$



**Fig. 2** Field vectors of the incident, transmitted, and reflected waves in case the electric field vectors are perpendicular to the plane of incidence (S polarization).

$$t_s = \frac{2(n_1/\mu_1) \cos \theta_i}{(n_1/\mu_1) \cos \theta_i + (n_2/\mu_2) \cos \theta_t} \quad (13)$$

Eqs. 7 and 8, as well as Eqs. 12 and 13 present Fresnel equations in their general form, which is also valid for materials with negative indices of refraction (also known as metamaterials or left-handed materials). When applying these equations to such materials, absolute values of the refractive indices must be used.<sup>[1-3]</sup>

Most commonly used optical materials are non-magnetic, so one can approximate  $\mu_1 = \mu_2 = \mu_0$ . Under this approximation, the permeabilities in Eqs. 7, 8, 12, and 13 cancel, and the Fresnel equations can be further simplified by incorporating Snell's law:

$$r_p = -\frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)} \quad (14)$$

$$t_p = \frac{2 \sin \theta_i \cos \theta_t}{\sin(\theta_i + \theta_t) \cos(\theta_i - \theta_t)} \quad (15)$$

$$r_s = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)} \quad (16)$$

$$t_s = \frac{2 \sin \theta_i \cos \theta_t}{\sin(\theta_i + \theta_t)} \quad (17)$$

We now proceed toward discussing the main consequences of the Fresnel equations.

## CONSEQUENCES AND SPECIAL CASES

### Intensity Reflectivity and Transmissivity

For most practical purposes, the reflection and transmission coefficients for the intensity, rather than field amplitudes, are of interest. For a wave of amplitude  $E$  propagating in a non-magnetic medium with the refractive index  $n$ , we have:

$$I = 2nc\epsilon_0 |E|^2 \quad (18)$$

where  $c$  is the speed of light in vacuum and  $\epsilon_0$  is the electric constant. Because the incident and reflected waves propagate

in the same medium, we can write for the intensity reflection coefficient:

$$R = \frac{|E_r|^2}{|E_i|^2} = |r|^2 \quad (19)$$

and thus,

$$R_p = \frac{\tan^2(\theta_i - \theta_t)}{\tan^2(\theta_i + \theta_t)} \quad (20)$$

$$R_s = \frac{\sin^2(\theta_i - \theta_t)}{\sin^2(\theta_i + \theta_t)} \quad (21)$$

From these, we obtain intensity transmissivities as follows:

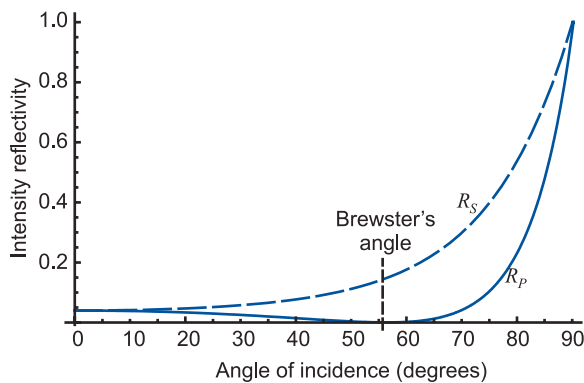
$$T_p = 1 - R_p = \frac{4 \sin \theta_i \sin \theta_t \cos \theta_i \cos \theta_t}{\sin^2(\theta_i + \theta_t) \cos^2(\theta_i - \theta_t)} \quad (22)$$

$$T_s = 1 - R_s = \frac{4 \sin \theta_i \sin \theta_t \cos \theta_i \cos \theta_t}{\sin^2(\theta_i + \theta_t)} \quad (23)$$

Note that, in contrast to the reflection coefficient, the intensity transmissivity is not simply the square of the amplitude transmissivity, as two additional factors must be taken into account. First, one must account for the refractive index of the propagation medium, which enters the expression for the intensity (Eq. 18). Second, the intensity is calculated per unit of the wavefront area, and the wavefronts of the incident and transmitted wave are tilted with respect to the interface at different angles  $\theta_i$  and  $\theta_t$ , respectively. Therefore, the intensity transmissivity is given by

$$T = \frac{n_2 \cos \theta_t |E_t|^2}{n_1 \cos \theta_i |E_i|^2} = \frac{n_2 \cos \theta_t}{n_1 \cos \theta_i} |t|^2 \quad (24)$$

A graph of the reflectivities (Eqs. 20, 21) for the vacuum–glass interface as a function of the angle of incidence is illustrated in Fig. 3. At normal incidence, the S- and P-polarized



**Fig. 3** Intensity reflectivities of the S- and P-polarization components at the interface of vacuum and glass with the index of refraction of 1.5.

waves are physically identical and have the same reflectivity of about 4%. At an incidence angle of  $\theta_i = 90^\circ$ , all of the incident light is reflected, so the interface acts as a mirror.

### Brewster's Effect

By analyzing Eq. 20 and Fig. 3, we observe that the reflectivity for the wave polarized in the plane of incidence vanishes when  $\theta_i + \theta_t = 90^\circ$ , so the denominator in the right-hand side of Eq. 20 becomes infinite. At this point, all incident light that is polarized parallel to the plane of incidence is transmitted. If the incident wave has both polarization components (or its polarization is random), the reflected wave is completely S-polarized.

The value of the angle of incidence at which this occurs is known as Brewster's angle  $\theta_B$ .

Writing Snell's law at Brewster's angle:

$$n_1 \sin \theta_B = n_2 \sin \theta_t = n_2 \sin\left(\frac{\pi}{2} - \theta_B\right) = n_2 \cos \theta_B \quad (25)$$

we find an explicit expression for that angle:

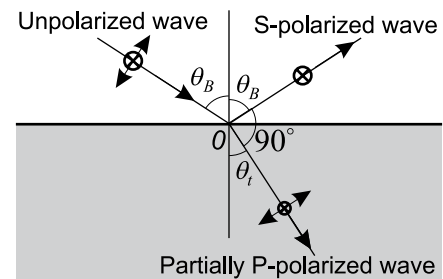
$$\tan \theta_B = \frac{n_2}{n_1} \quad (26)$$

which is referred to as *Brewster's law*.

Brewster's law may be understood by the following intuitive argument (Fig. 4). Consider an interface between vacuum and glass. The reflected wave is generated by elementary molecular dipoles inside the glass that are excited by the transmitted wave. These oscillations are parallel to the electric field in this wave. But when the transmitted and reflected wave vectors are directed at a right angle to each other, the electric field in the transmitted P-polarized wave, and hence the elementary dipoles inside the glass, oscillate parallel to  $\vec{k}_r$ . Hence, the dipoles would have to excite a wave propagating in the same direction as the direction of their oscillation, and this is impossible because the electromagnetic wave is transverse.

### Phase of the Reflected Wave

For the direction of the incident wave close to normal, we find the amplitude reflectivities (Eqs. 14 and 16) to be negative if  $n_1 < n_2$  (if the sign convention of Figs. 1 and 2 is used).



**Fig. 4** Reflection and transmission at Brewster's angle. The arrows correspond to the electric field vectors.

This implies that the phase of the wave shifts by  $180^\circ$  when reflection from a medium with a higher index of refraction occurs. For the S-polarization case, the amplitude reflectivity has the same sign for all incidence angles; for the P-polarization, it changes sign when the angle of incidence exceeds Brewster's angle.

Because the amplitude transmission coefficients are always positive, the transmitted wave does not experience any phase shift with respect to the incident wave.

### Total Internal Reflection

Another phenomenon that can be derived from examining the Fresnel equations is the phase shift of the wave that has undergone total internal reflection. Total internal reflection occurs when  $n_1 > n_2$  and  $(n_1/n_2)\sin\theta_i > 1$ ; thus, Snell's law cannot hold. The result is that the incident wave is totally reflected and the transmitted wave is of evanescent rather than plane wave character. Since the behavior of the evanescent wave is largely counterintuitive, it is instructive to briefly summarize its properties before proceeding to modify the Fresnel equations for situations involving such waves.

The spatiotemporal behavior of the electric field in this wave can be written as

$$\vec{E}(\vec{r}, t) = \vec{E}_t e^{i\vec{k}_t \cdot \vec{r} - i\omega t} + c.c. \quad (27)$$

where  $\omega$  is the angular frequency,  $\vec{k}_t$  is the wave vector, and *c.c.* refers to the complex conjugate term. The component of the wave vector that is parallel to the interface must be the same for the incident and transmitted waves:  $(k_t)_x = (k_i)_x = (\omega/c)n_1 \sin\theta_i$ . Since the evanescent wave must comply with the wave equation:

$$\nabla^2 \vec{E}(\vec{r}, t) = (n_2^2/c^2) \ddot{\vec{E}}(\vec{r}, t) \quad (28)$$

we find that  $(k_t)_x^2 + (k_t)_z^2 = n_2^2 \omega^2/c^2$ ; thus, the component of the transmitted wave vector that is normal to the interface is imaginary:

$$(k_t)_z = i \frac{\omega}{c} \sqrt{n_1^2 \sin^2 \theta_i - n_2^2} \quad (29)$$

This is not surprising because substituting an imaginary  $(k_t)_z$  into Eq. 27, we obtain a wave that decays exponentially with the distance from the interface, as expected from an evanescent wave. We thus find,

$$\vec{k}_t = n_2 \frac{\omega}{c} (S, 0, iC) \quad (30)$$

where  $S = (n_1/n_2) \sin\theta_i$  and

$$C = \sqrt{S^2 - 1} \quad (31)$$

Knowing the wave vector components, we can determine the components of the electric field amplitude vector in the transmitted wave using Gauss's law  $\vec{\nabla} \cdot \vec{E}(\vec{r}, t) = 0$ , which

we rewrite using Eq. 27 as  $\vec{k}_t \cdot \vec{E}_t = 0$ . Accordingly, we find that

$$\vec{E}_t = E_t (-C, 0, -iS) \quad (32)$$

for the P-polarization case (where we assumed, as previously, that the x-component of the transmitted electric field vector is real and negative) and

$$\vec{E}_t = E_t (0, 1, 0) \quad (33)$$

for the S-polarization.

Now by applying Faraday's law  $\vec{\nabla} \times \vec{E}(\vec{r}, t) = -\mu \dot{\vec{H}}(\vec{r}, t)$  to Eq. 27, we find for the electric and magnetic field amplitudes of the transmitted wave:

$$\vec{k} \times \vec{E}_t = \mu \omega \vec{H}_t \quad (34)$$

and thus,

$$\vec{H}_t = E_t \frac{n_2}{c\mu_2} (0, i, 0) \quad (35)$$

for the P-polarization case and

$$\vec{H}_t = E_t \frac{n_2}{c\mu_2} (-iC, 0, S) \quad (36)$$

for the S-polarization. Note that Eq. 4, which we used for plane waves, is not applicable to evanescent waves.

Equalizing the x- and y-components of the electric and magnetic field amplitudes above and below the surface, we obtain the Fresnel equations for the case of total internal reflection:

$$r_P = -\frac{(n_2^2/\mu_2) \cos\theta_i - i(n_1/\mu_1) \sqrt{n_1^2 \sin^2 \theta_i - n_2^2}}{(n_2^2/\mu_2) \cos\theta_i + i(n_1/\mu_1) \sqrt{n_1^2 \sin^2 \theta_i - n_2^2}} \quad (37)$$

$$r_S = \frac{(n_1/\mu_1) \cos\theta_i - (i/\mu_2) \sqrt{n_1^2 \sin^2 \theta_i - n_2^2}}{(n_1/\mu_1) \cos\theta_i + (i/\mu_2) \sqrt{n_1^2 \sin^2 \theta_i - n_2^2}} \quad (38)$$

These results can be interpreted as follows. In the case of regular refraction, the z-component of the transmitted wave vector equals

$$(k_t)_z = n_2 (\omega/c) \cos\theta_t = n_2 (\omega/c) \sqrt{1 - \frac{n_1^2}{n_2^2} \sin^2 \theta_i} \quad (39)$$

For total internal reflection, this component becomes complex, so one can formally write:

$$\cos\theta_t = iC = i \sqrt{\frac{n_1^2}{n_2^2} \sin^2 \theta_i - 1} \quad (40)$$

Substituting this expression into Eqs. 7 and 12, one obtains Eqs. 37 and 38, respectively.

The absolute values of the numerators and denominators of Eqs. 37 and 38 are equal; thus we find for total internal reflection, that  $R_p = R_s = 1$ , in accordance with Eq. 19. On the other hand, the amplitude reflectivity being a complex number implies that the reflected wave experiences an optical phase shift with respect to the incident wave, which is given by

$$\tan \frac{\delta_p - \pi}{2} = -\frac{\mu_2}{\mu_1} \frac{n_1 \sqrt{n_1^2 \sin^2 \theta_i - n_2^2}}{n_2^2 \cos \theta_i} \quad (41)$$

$$\tan \frac{\delta_s}{2} = -\frac{\mu_1}{\mu_2} \frac{\sqrt{n_1^2 \sin^2 \theta_i - n_2^2}}{n_1 \cos \theta_i} \quad (42)$$

with the zero phase corresponding to the vector orientations defined in Figs. 1 and 2. We see that these phase shifts are different for the S- and P-polarized waves. In other words, a linearly polarized wave will generally be elliptically polarized after it has experienced total internal reflection.

If medium 2 is a metamaterial, the associated phase shifts are opposite with respect to those obtained in reflection from a right-handed material with the same magnitudes of  $n_2$  and  $\mu_2$ .<sup>[2,3]</sup>

### Goos-Hänchen shift

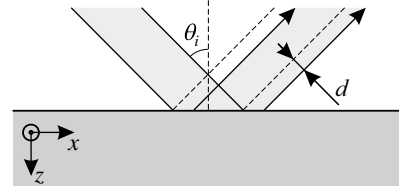
An important consequence of the phase shift associated with the total internal reflection is the spatial displacement experienced by an optical beam undergoing such reflection, known as the Goos-Hänchen shift (Fig. 5).<sup>[4]</sup> This phenomenon can be understood by analyzing the spatial distribution of the incident field amplitude in the interface plane,  $\vec{E}_i(x)$  and its Fourier transform over  $x$ , given by  $\vec{E}(k_x)$ , such that

$$\vec{E}(x) = \int_{-\infty}^{+\infty} \vec{E}(k_x) e^{ik_x x} dk_x \quad (43)$$

(where we neglect the dependence of the field on  $y$ , which plays no role in this argument). In other words, we consider the incident field as a sum of infinitely many plane waves, each having a slightly different  $x$  component of its wave vector, and hence, a slightly different angle of propagation  $\theta_i(k_x) = \arcsin(k_x/k_i)$ . Accordingly, in total internal reflection, each of these plane waves experiences a different phase shift, which can be decomposed into the first-order Taylor series as

$$\delta(k_x) \approx \delta(k_{x0}) + \left. \frac{d\delta(k_x)}{dk_x} \right|_{k_{x0}} (k_x - k_{x0}) \quad (44)$$

where  $k_{x0}$  corresponds to the direction of the incident beam axis. Eq. 44 is valid if the beam diameter greatly exceeds the wavelength; thus, the relevant range of values of  $\Delta k_x$  is small. For the reflected wave, we then obtain:



**Fig. 5** Good-Hänchen effect (the totally internally reflected beam undergoes a spatial dislocation shift by a small distance  $d$  with respect to the position of its specular reflection, illustrated by dashed lines).

$$\begin{aligned} \vec{E}_r(x) &= \int_{-\infty}^{+\infty} \vec{E}(k_x) e^{i[k_x x + \delta(k_x)]} dk_x \\ &\approx e^{i\delta(k_{x0})} \int_{-\infty}^{+\infty} \vec{E}(k_x) e^{ik_x(x + d\delta/dk_x)} dk_x \\ &= e^{i\delta(k_{x0})} \vec{E}_i \left( x + \frac{d\delta}{dk_x} \right) \end{aligned} \quad (45)$$

Neglecting the constant phase factor, we find that the reflected wave is spatially displaced with respect to the incident one. The lateral displacement of the reflected beam is then obtained as (Fig. 5):

$$d = -d\delta/dk_x \cos \theta_i \quad (46)$$

Substituting Eqs. 41 and 42 into the foregoing result and keeping in mind that  $k_x = k_i \sin \theta_i$ , we find the expressions for the Goos-Hänchen shift in the two polarizations.<sup>[2,3]</sup>

$$\begin{aligned} d_p &= \frac{2}{k_1} \frac{n_2^2}{n_1^2} \frac{\mu_1}{\mu_2} \frac{1}{\sqrt{\sin^2 \theta_i - \frac{n_2^2}{n_1^2}}} \\ &\quad \times \frac{\left(1 - \frac{n_2^2}{n_1^2}\right) \sin \theta_i}{\left[\left(\frac{n_2}{n_1}\right)^4 \left(\frac{\mu_1}{\mu_2}\right)^2 \cos^2 \theta_i + \sin^2 \theta_i - \frac{n_2^2}{n_1^2}\right]} \end{aligned} \quad (47)$$

$$d_s = \frac{2}{k_1} \frac{\mu_2}{\mu_1} \frac{\left(1 - \frac{n_2^2}{n_1^2}\right) \sin \theta_i}{\left[\left(\frac{\mu_2}{\mu_1}\right)^2 \cos^2 \theta_i + \sin^2 \theta_i - \frac{n_2^2}{n_1^2}\right] \sqrt{\sin^2 \theta_i - \frac{n_2^2}{n_1^2}}} \quad (48)$$

where  $k_1 = \omega n_1/c$ . As seen from the foregoing equations, at incidence angles that are significantly larger than the critical angle, the Goos-Hänchen shift is on a scale of the optical wavelength. For right-handed materials, it is always in the positive  $x$  direction. This can be visualized using the ray picture of light: in total internal reflection, the incident rays bounce not off the interface, but slightly below the

interface, accounting for the existence of the evanescent wave. However, if medium 2 is a metamaterial, the Goos-Hänchen shift is in the negative  $x$  direction due to counter-intuitive direction of refraction in metamaterials.<sup>[2,3]</sup>

## APPLICATIONS

One of the primary consequences of the Fresnel equations is that any interface between transparent optical media results in a significant fraction of the light being reflected. This is particularly important for complex lens systems such as microscope, telescope, and camera objectives. Given that the spurious reflectivity at a single glass-air interface is 4%, a system of 8 optical elements will suffer from about 50% loss due to Fresnel reflections.

To avoid these losses, antireflection coatings are commonly used in lens systems. In fiber optics, an alternative solution is offered by index-matching materials: liquid or gel substances whose index of refraction approximates that of the fiber core. Placing an index-matching fluid in fiber connectors and mechanical splices greatly reduces Fresnel reflection at the surfaces and thus decreases the power loss.

Brewster's effect is extensively used in photography. Unpolarized light, incident on a building window or water surface, becomes largely S-polarized after reflection. Dependent on the orientation of a polarizing filter in front of the camera, the amount of the reflected light can be regulated. In particular, aligning this filter to transmit only the P polarization permits taking pictures of objects beneath the surface or behind the window.

Polarizing sunglasses provide another example of practical application of Brewster's effect. These sunglasses are designed to block horizontal polarization, which helps reducing glare from horizontal objects such as water or road surfaces.

A further application of Brewster's effect is found in laser physics, specifically in gas laser design. The end windows of laser tubes are routinely manufactured to be oriented at the Brewster angle with respect to the cavity mode, with an aim to eliminate reflection losses in the P-polarization. In this way, a stronger gain per cavity roundtrip can be achieved for one of the polarization components while reducing the gain for the other. This helps in obtaining strong emission in a single polarization mode.

An interesting application of Fresnel equations was proposed by Fresnel himself. As mentioned earlier, total internal reflection causes different phase shifts to the S- and P-polarized components of the incident wave. Fresnel used this phenomenon to design an optical element that converts light polarization from linear into circular. This is accomplished by means of two total internal reflections in a parallelepiped prism, as illustrated in Fig. 6. For a prism made of glass with a refractive index of 1.5, an internal reflection angle of incidence of  $54.6^\circ$  can be used. It should be noted that at present, polarization transformations in free space are typically performed by birefringent waveplates rather than the Fresnel rhomb. This is because waveplates are more compact and do not distort the beam position.

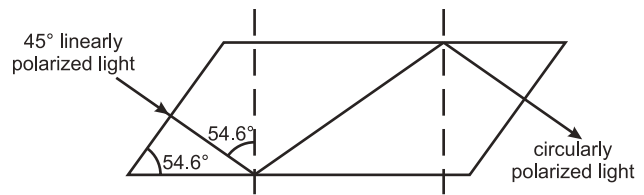


Fig. 6 Fresnel rhomb.

## CONCLUSION

We have derived the Fresnel equations from the first principles of wave optics. Subsequently, we discussed the consequences of these equations, such as the Brewster effect and the optical phase shift in partial and total internal reflection. Finally, we discussed a few applications of the Fresnel equations and the related effects in optical design.

## HISTORICAL NOTES

Augustin-Jean Fresnel (1788–1827) is one of the founding fathers of the wave theory of light. In response to an 1818 competition held by the French Academy of Sciences, Fresnel wrote a memoir describing diffraction as a wave phenomenon. Although the corpuscular (Newtonian) concept of light was universally accepted at that time, Fresnel's theory received immediate experimental confirmation, thus revolutionizing contemporary optical science. In 1823, Fresnel was unanimously elected a member of the Academy, and in 1825 he became a member of the Royal Society of London. At that time, Fresnel developed his theory based on the theory of elastic ether. In 1827, the Royal Society of London awarded him the Rumford Medal.

Sir David Brewster (1781–1868) is mostly remembered for his invention of the kaleidoscope and optical improvements of the microscope. However, his main experiments were on the theory of light and its uses. His first paper, "Some Properties of Light," was published in 1813. Brewster's Law was named after him in 1814 when he made measurements on the angle of maximum polarization using biaxial crystals. He was awarded all three of the principal medals of the Royal Society for his optical research (Copley medal, 1815; Rumford medal, 1818; Royal medal, 1830). He was also knighted in 1831.

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